Supplementary Material Titled “Discrimination-Based Double Auction for Maximizing Social Welfare in the Electricity and Heating Market Considering Privacy Preservation”

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**Proof of Lemma 2:** Assuming that *a* is a NE, holds for all entity *j*. Otherwise, assuming that for a certain entity *j*, its utility function:

 (1)

When there is a discontinuity at , ESP *j* can always adjust the bid to improve the utility, which contradicts *a* is the NE. Therefore, is established for all ESP *j*, and the utility function of ESP is:

(2)

It is continuous at , when , only consider , because when , the payment from ETC to ESP is , ESP gains negative utility. Therefore, the utility function at is strictly concave.

When *J=2*, there is no Nash equilibrium. The method of reproof, assuming that is Nash equilibrium, the utility function of ESP is:

 (3)

Regarding monotonous increase, ESP1 can unilaterally increase bidding to obtain higher utility, and there is no Nash equilibrium.

**Proof of Theorem 2:** *Existence*:1) is a non-empty, convex, and compact subset of some Euclidean space. 2) is strictly concave. The optimal solution is *,*and the optimization problem on the supply side satisfies the Slater condition [26], there is a unique Lagrange multiplier that meets the following KKT conditions:

 (4)

Satisfy the condition of Lemma 2, *a* is the supply side NE.

*Uniqueness*: Suppose *a* is the NE on the supply side, known from Lemma 2:

 (5)

*s* is the corresponding energy supply plan and the KKT condition of the supply-side optimization problem. With and , this shows that *a* is unique.

The above proves that at least two positive components exist and unique for the supply quantity *s* corresponding to the Nash equilibrium *a.* When the supply energy *s* corresponding to the Nash equilibrium has only one positive component, at this time, assume that ESP1 supplies all the energy, , and there are an infinite number of *v* satisfying Lemma 2:

 (6)

The minimum value is . At this time, the Nash equilibrium on the supply side is , where:

 (7)

All Nash equilibriums supply the same energy *s.*